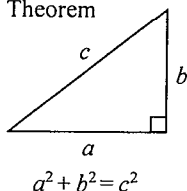


**IGCSE MATHEMATICS 4400
FORMULA SHEET – HIGHER TIER**

Pythagoras' Theorem

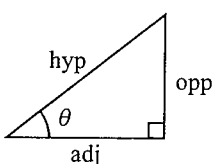
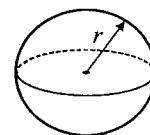
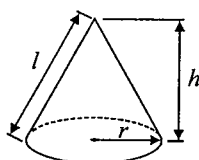


Volume of cone = $\frac{1}{3}\pi r^2 h$

Volume of sphere = $\frac{4}{3}\pi r^3$

Curved surface area of cone = $\pi r l$

Surface area of sphere = $4\pi r^2$



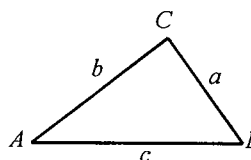
adj = hyp \times cos θ
opp = hyp \times sin θ
opp = adj \times tan θ

or $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

$\cos \theta = \frac{\text{adj}}{\text{hyp}}$

$\tan \theta = \frac{\text{opp}}{\text{adj}}$

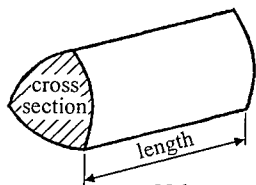
In any triangle ABC



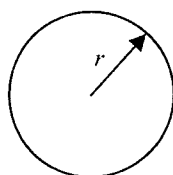
Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2} ab \sin C$



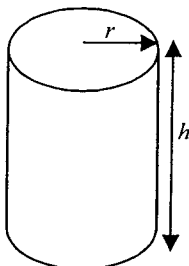
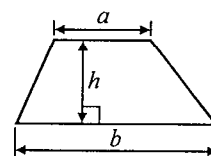
Volume of prism = area of cross section \times length



Circumference of circle = $2\pi r$

Area of circle = πr^2

Area of a trapezium = $\frac{1}{2}(a + b)h$



Volume of cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2\pi r h$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Leave
blank

Answer ALL NINETEEN questions.

Write your answers in the spaces provided.

You must write down all stages in your working.

1. (a) Use your calculator to work out the value of

$$\frac{(3.7+4.6)^2}{2.8+6.3}$$

Write down all the figures on your calculator display.

7.57032967
.....
(2)

- (b) Give your answer to part (a) correct to 2 decimal places.

7.57
.....
(1)

Q1

(Total 3 marks)

2. (a) Work out the value of $x^2 - 5x$ when $x = -3$

$$9 - -15$$

24
.....
(2)

- (b) Factorise $x^2 - 5x$

$x(x-5)$
.....
(2)

Q2

(Total 4 marks)

3

Turn over



N 2 5 7 9 9 A 0 3 2 0

Leave
blank

3. Hajra counted the numbers of sweets in 20 packets.
The table shows information about her results.

Number of sweets	Frequency
46	3
47	6
48	3
49	5
50	2
51	1

Work out the mean number of sweets in the 20 packets.

$$46 \times 3 + 47 \times 6 + \dots = 960$$

$$3 + 6 + 3 + \dots = 20$$

$$\frac{960}{20}$$

48

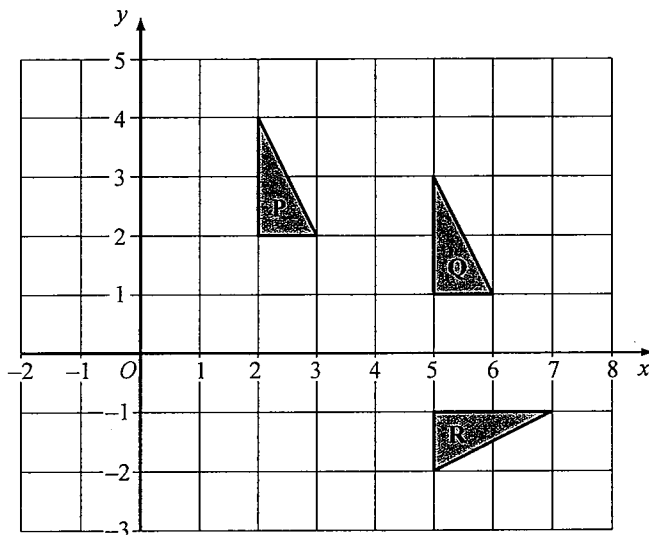
(Total 3 marks)

Q3



Leave blank

4.



- (a) Describe fully the single transformation which maps triangle P onto triangle Q.

..... Translate by $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$
.....
(2)

- (b) Describe fully the single transformation which maps triangle P onto triangle R.

..... Rotation of 90° clockwise about $(2, -1)$
.....
(3)

(Total 5 marks)

Q4



N 2 5 7 9 9 A 0 5 2 0

Leave blank

5. (a) Simplify, leaving your answers in index form,

(i) $7^5 \times 7^3$

7^8

(ii) $5^9 \div 5^3$

5^6
(2)

(b) Solve $\frac{2^9 \times 2^4}{2^n} = 2^8$

$$\frac{2^{13}}{2^n} = 2^8$$
$$13 - n = 8$$

$n = 5$
(2)

(Total 4 marks)

Q5

6. (a) Expand and simplify $3(4x - 5) - 4(2x + 1)$

$$12x - 15 - 8x - 4$$

$4x - 19$
(2)

(b) Expand and simplify $(y + 8)(y + 3)$

$$y^2 + 3y + 8y + 24$$

$y^2 + 11y + 24$
(2)

(c) Expand $p(5p^2 + 4)$

$5p^3 + 4p$
(2)

(Total 6 marks)

Q6



Leave
blank

7. A tunnel is 38.5 km long.

(a) A train travels the 38.5 km in 21 minutes.

Work out the average speed of the train.
Give your answer in km/h.

$$S = \frac{D}{T} = \frac{38.5}{\left(\frac{21}{60}\right)}$$

..... 110 km/h
(3)

(b) To make the tunnel, a cylindrical hole 38.5 km long was drilled.
The radius of the cylindrical hole was 4.19 m.

Work out the volume of earth, in m^3 , which was removed to make the hole.
Give your answer correct to 3 significant figures.

$$\begin{aligned} \pi r^2 h &= \pi \times 4.19^2 \times 38500 \\ &= 2123433.4.. \end{aligned}$$

..... 2120000 m^3
(3)

(Total 6 marks)

Q7



Leave
blank

8. (a) Shri invested 4500 dollars. After one year, he received 270 dollars interest.
Work out 270 as a percentage of 4500

$$\frac{270}{4500} = 0.06$$

.....6..... %
(2)

- (b) Kareena invested an amount of money at an interest rate of 4.5% per year.
After one year, she received 117 dollars interest.
Work out the amount of money Kareena invested.

$$x \times 0.045 = 117$$

$$x = \frac{117}{0.045}$$

.....2600..... dollars
(2)

- (c) Ravi invested an amount of money at an interest rate of 4% per year.
At the end of one year, interest was added to his account and the total amount in his
account was then 3328 dollars.
Work out the amount of money Ravi invested.

$$x \times 1.04 = 3328$$

$$x = \frac{3328}{1.04}$$

.....3200..... dollars
(3)

(Total 7 marks)

Q8



Leave
blank

9. (a) Solve $5x - 4 = 2x + 7$

$$3x = 11$$
$$\Rightarrow x = \frac{11}{3}$$

$$x = \dots\dots\dots 3\frac{2}{3} \dots\dots\dots$$

(2)

(b) Solve $\frac{7-2y}{4} = 2y+3$

$$7-2y = 4(2y+3)$$
$$= 8y+12$$

$$\Rightarrow -5 = 10y$$

$$\Rightarrow y = \frac{-5}{10} = -\frac{1}{2}$$

$$y = \dots\dots\dots -\frac{1}{2} \dots\dots\dots$$

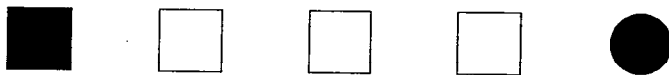
(4)

(Total 6 marks)

Q9



10. Here are five shapes.



Four of the shapes are squares and one of the shapes is a circle.
 One square is black.
 Three squares are white.
 The circle is black.
 The five shapes are put in a bag.

- (a) Jasmine takes a shape at random from the bag 150 times.
 She replaces the shape each time.

Work out an estimate for the number of times she will take a white square.

$$150 \times \frac{3}{5}$$

$$\dots\dots\dots 90$$

(3)

- (b) Alec takes a shape at random from the bag and does **not** replace it.
 Bashir then takes a shape at random from the bag.

Work out the probability that

- (i) they both take a square,

$$\frac{4}{5} \text{ S } \frac{3}{4} \text{ S} \quad \frac{4}{5} \times \frac{3}{4} =$$

$$\dots\dots\dots \frac{3}{5}$$

- (ii) they take shapes of the same colour.

$$\begin{array}{l} \frac{2}{5} \text{ B } \frac{1}{4} \text{ B} \\ \frac{3}{5} \text{ W } \frac{2}{4} \text{ W} \end{array} \quad \frac{2}{5} \times \frac{1}{4} + \frac{3}{5} \times \frac{2}{4}$$

$$\dots\dots\dots \frac{2}{5}$$

(5)

Q10

(Total 8 marks)



11.

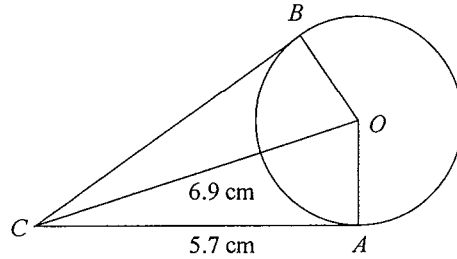


Diagram NOT accurately drawn

A and B are points on a circle, centre O .
 The lines CA and CB are tangents to the circle.
 $CA = 5.7$ cm.
 $CO = 6.9$ cm.

- (a) Give a reason why angle $CAO = 90^\circ$.

A tangent and radius at a point are perpendicular

(1)

- (b) Calculate the perimeter of the kite $CAOB$.
 Give your answer correct to 3 significant figures.

$$OA = \sqrt{6.9^2 - 5.7^2}$$

$$= 3.8884\dots$$

$$\text{Perim} = 2 \times 5.7 + 2 \times 3.8884$$

$$= 19.176\dots$$

19.2 cm
 (5)

(Total 6 marks)

Q11



12. The grouped frequency table gives information about the weights of 60 cows.

Weight (w kg)	Frequency
$100 < w \leq 200$	10
$200 < w \leq 300$	16
$300 < w \leq 400$	15
$400 < w \leq 500$	9
$500 < w \leq 600$	6
$600 < w \leq 700$	4

(a) Complete the cumulative frequency table.

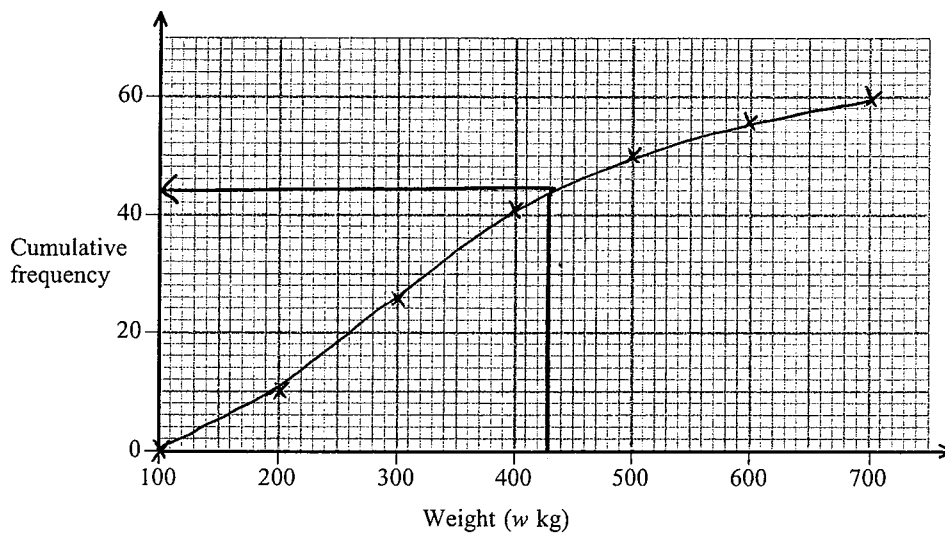
Weight (w kg)	Cumulative frequency
$100 < w \leq 200$	10
$100 < w \leq 300$	26
$100 < w \leq 400$	41
$100 < w \leq 500$	50
$100 < w \leq 600$	56
$100 < w \leq 700$	60

(1)



Leave blank

(b) On the grid, draw the cumulative frequency graph for your table.



(2)

(c) Use your graph to find an estimate for the number of cows that weighed more than 430 kg.
Show your method clearly.

*44 weigh less
ie. $60 - 44 = 16$ weigh more*

16

(2)

Q12

(Total 5 marks)



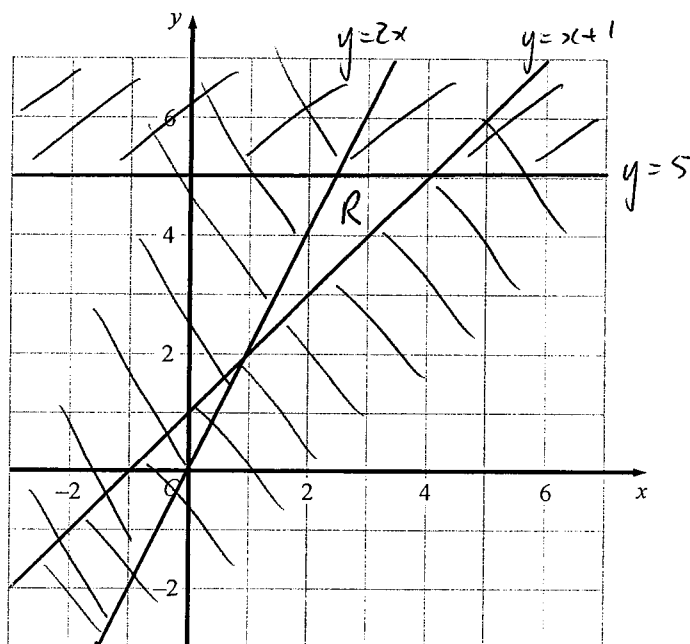
N 2 5 7 9 9 A 0 1 3 2 0

Leave blank

13. Show, by shading on the grid, the region which satisfies all three of these inequalities.

$$y \leq 5 \quad y \leq 2x \quad y \geq x + 1$$

Label your region **R**.



(Total 4 marks)

Q13



14. (a) Make r the subject of the formula $A = \pi r^2$, where r is positive.

$$r = \sqrt{\frac{A}{\pi}} \dots\dots\dots (2)$$

The area of a circle is 14 cm^2 , correct to 2 significant figures.

(b) (i) Work out the lower bound for the radius of the circle.
Write down all the figures on your calculator display.

$$\sqrt{\frac{13.5}{\pi}} \leftarrow \text{l.b. of } 14$$

$$\dots\dots\dots 2.072964897 \text{ cm}$$

(ii) Give the radius of the circle to an appropriate degree of accuracy.
You must show working to explain how you obtained your answer.

$$\begin{aligned} \text{u.b.} &= \sqrt{\frac{14.5}{\pi}} \\ &= 2.14836\dots \end{aligned}$$

Both round to 2.1 to 1 d.p.
so the radius must be 2.1 to 1 d.p.

$$\dots\dots\dots 2.1 \text{ cm} \quad (4)$$

(Total 6 marks)

Q14



Leave blank

15. The frequency, f kilohertz, of a radio wave is inversely proportional to its wavelength, w metres.

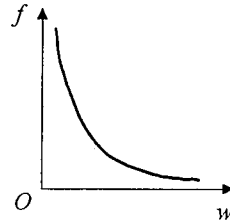
When $w = 200, f = 1500$

(a) (i) Express f in terms of w .

$$\begin{aligned} f &\propto \frac{1}{w} \\ f &= \frac{k}{w} \\ 1500 &= \frac{k}{200} \Rightarrow k = 300000 \end{aligned}$$

$$f = \frac{300000}{w}$$

(ii) On the axes, sketch the graph of f against w .



(4)

(b) The wavelength of a radio wave is 1250 m.
Calculate its frequency.

$$f = \frac{300000}{1250}$$

..... 240 kilohertz
(2)

(Total 6 marks)

Q15



16. PQR is a triangle.
 E is the point on PR such that $PR = 3PE$.
 F is the point on QR such that $QR = 3QF$.

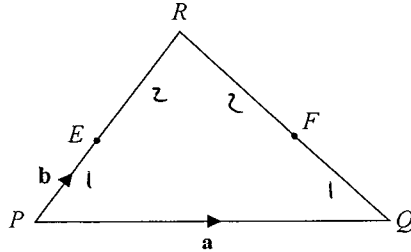


Diagram NOT accurately drawn

$$\vec{PQ} = \mathbf{a}, \quad \vec{PE} = \mathbf{b}.$$

- (a) Find, in terms of \mathbf{a} and \mathbf{b} ,

(i) \vec{PR}

3b

(ii) \vec{QR}

-a + 3b

3b - a

(iii) $\vec{PF} = \vec{PQ} + \vec{QF}$
 $= \mathbf{a} + \frac{1}{3}(3\mathbf{b} - \mathbf{a})$
 $= \mathbf{a} + \mathbf{b} - \frac{1}{3}\mathbf{a}$

$\frac{2}{3}\mathbf{a} + \mathbf{b}$ (3)

- (b) Show that $\vec{EF} = k\vec{PQ}$ where k is an integer.

$$\begin{aligned} \vec{EF} &= -\mathbf{b} + \frac{2}{3}\mathbf{a} + \mathbf{b} \\ &= \frac{2}{3}\mathbf{a} \\ &= \frac{2}{3}\vec{PQ} \\ &= k\vec{PQ} \text{ for } k = \frac{2}{3} \end{aligned}$$

(2)

Q16

(Total 5 marks)



Leave
blank

17. A curve has equation $y = x^2 + \frac{16}{x}$

The curve has one turning point.

Find $\frac{dy}{dx}$ and use your answer to find the coordinates of this turning point.

$$\begin{aligned}\frac{dy}{dx} &= 2x + 16x^{-2} \\ &= 2x - \frac{16}{x^2} = 0\end{aligned}$$

$$\Rightarrow 2x^3 - 16 = 0$$

$$\Rightarrow 2x^3 = 16$$

$$\Rightarrow x = 2$$

$$\begin{aligned}\Rightarrow y &= 2^2 + \frac{16}{2} \\ &= 12\end{aligned}$$

(2, 12)

(Total 4 marks)

Q17



18.

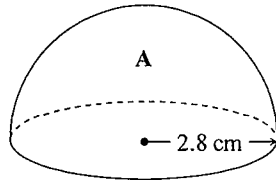


Diagram NOT accurately drawn

A solid hemisphere **A** has a radius of 2.8 cm.

- (a) Calculate the **total** surface area of hemisphere **A**.
Give your answer correct to 3 significant figures.

$$\begin{aligned} & \pi r^2 + \frac{1}{2} \times 4\pi r^2 \\ &= \pi \times 2.8^2 + 2 \times \pi \times 2.8^2 \\ &= 73.890\dots \end{aligned}$$

..... 73.9 cm²
(3)

A larger solid hemisphere **B** has a **volume** which is 125 times the volume of hemisphere **A**.

- (b) Calculate the **total** surface area of hemisphere **B**.
Give your answer correct to 3 significant figures.

$$\begin{aligned} \text{r.o.v.} &= 1:125 \\ \text{r.o.l.} &= 1:5 \leftarrow \sqrt[3]{125} \\ \text{r.o.a.} &= 1:25 \end{aligned}$$

So surface area of B

$$\begin{aligned} &= 25 \times 73.89\dots \\ &= 1847.25\dots \end{aligned}$$

..... 1850 cm²
(3)

(Total 6 marks)

Q18

PLEASE TURN OVER FOR QUESTION 19



19. Solve the simultaneous equations

$$y = 3x - 1$$

$$x^2 + y^2 = 5$$

$$x^2 + (3x-1)^2 = 5$$

$$\Rightarrow x^2 + (3x-1)(3x-1) = 5$$

$$\Rightarrow x^2 + 9x^2 - 6x + 1 = 5$$

$$\Rightarrow 10x^2 - 6x - 4 = 0$$

$$\Rightarrow 5x^2 - 3x - 2 = 0$$

$$\Rightarrow x = \frac{3 \pm \sqrt{49}}{10}$$

$$= 1 \text{ or } -0.4$$

$$\text{+ } y = 2 \text{ or } -2.2$$

$$\left(-\frac{2}{5}, -2\frac{1}{5}\right)$$

$$(1, 2) \text{ or } (-0.4, -2.2)$$

Q19

(Total 6 marks)

TOTAL FOR PAPER: 100 MARKS

END

