MEI

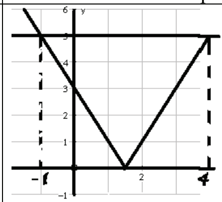
Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

METHODS OF ADVANCED MATHEMATICS, C3

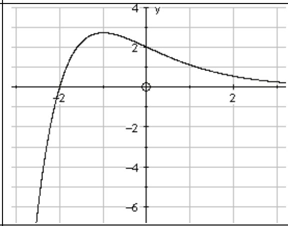
Practice Paper C3-B

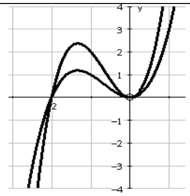
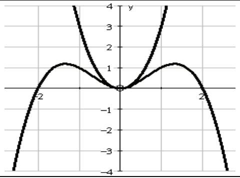
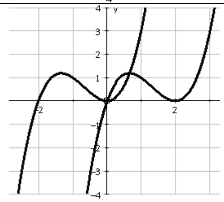
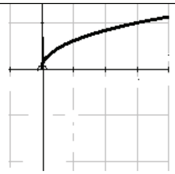
MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1		Call the numbers n, $n+1$ and $n+2$ At least one of the numbers is even, and so the product is a multiple of 2. If n is a multiple of 3 then so is the product. If $n = 3k + 1$ then $n + 2$ is a multiple of 3 If $n = 3k + 2$ then $n + 1$ is a multiple of 3. n must have one of the forms $3k$, $3k + 1$ or $3k + 2$. Therefore whichever it is one of the three numbers is a multiple of 3 and so the product is a multiple of 3. Since it is also a multiple of 2 it is a multiple of 6.	B1 M1 M1 E1 **4**	Algebra Divisibility by 2 Divisibility by 3 conclusion
2	**(i)**		B1 B1 **2**	Right part Left part
	(ii)	Line $y = 5$ to be shown on graph. $-1 < x < 4$	M1 A1 **2**	
3	**(i)**	$y = \left(x^2+3\right)^5$ Let $u = x^2 + 3 \Rightarrow \frac{du}{dx} = 2x$ $y = u^5 \Rightarrow \frac{dy}{du} = 5u^4$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5u^4 \times 2x = 10x\left(x^2+3\right)^4$	M1 A1 A1 **3**	Chain rule $\frac{dy}{du}$
	(ii)	$y = \frac{\sin 2x}{x}$ Let $u = \sin 2x \Rightarrow \frac{du}{dx} = 2\cos 2x$ $v = x \Rightarrow \frac{dv}{dx} = 1$ $\Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{2x\cos 2x - \sin 2x}{x^2}$	M1 A1 A1 **3**	Quotient rule
4		$y^2 = 5x - 4 \Rightarrow 2y\frac{dy}{dx} = 5 \Rightarrow \frac{dy}{dx} = \frac{5}{2y}$ When $x = 8, y^2 = 36 \Rightarrow y = \pm 6$ $\Rightarrow$ gradients $= \frac{5}{12}$ and $-\frac{5}{12}$	M1 A1 A1 A1 A1 **5**	

5		$x = x_0 e^{-3t} \Rightarrow e^{3t} = \frac{x_0}{x}$ $\Rightarrow 3t = \ln\left(\frac{x_0}{x}\right) \Rightarrow t = \frac{1}{3}\ln\left(\frac{x_0}{x}\right)$ $\Rightarrow t = \ln\left(\frac{x_0}{x}\right)^{\frac{1}{3}}$ i.e. $a = x_0,\ b = \frac{1}{3}$	M1 A1 A1 A1 **4**	Take logs or any equivalent method
6	**(i)**	$\int (2x-3)^7\,dx$. Let $u = 2x-3$, $\frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}du$ $= \int \frac{1}{2}u^7\,du = \frac{u^8}{2\times 8} = \frac{1}{16}(2x-3)^8 + c$	M1 A1 A1 **3**	or B3 cao
	(ii)	The substitution $u = x^2+1$ gives $\frac{du}{dx} = 2x$ $\Rightarrow \int_1^2 x(x^2+1)^3\,dx \quad = \int_2^5 \frac{1}{2}u^3\,du$ $= \left[\frac{u^4}{8}\right]_2^5$ $= \frac{609}{8}\ (= 76\tfrac{1}{8})$	M1 A1 A1 A1 A1 **5**	Using sub Correct int Correct limits Int Ans
7	**(i)**	$f^2(x) = 4x$	B1 **1**	
	(ii)	$fgh(x) = fg(x+2)$ $= f(x+2)^2$ $= 2(x+2)^2$	M1 A1 A1 **3**	correct order of functions
	(iii)	$y = h(x)$ $= x+2$ $\Rightarrow x = y-2$ $h^{-1}(x) = x-2$	B1 **1**	

Section B

8	**(i)**	$0 = (x+2)e^{-x}$ $\Rightarrow x = -2$ so (-2,0) and (0,2)	B1 B1 **2**	
	(ii)	$y = (x+2)e^{-x}$ $\Rightarrow \frac{dy}{dx} = -e^{-x}(x+1) = 0 \Rightarrow x = -1$ SP is (-1,e)	M1 A1 M1 A1 **4**	Product rule = 0
	(iii)	$\Rightarrow \frac{d^2y}{dx^2} = xe^{-x}$ At (-1,e) this is negative, so SP is a maximum.	M1 A1 A1 **3**	
	(iv)		B1 **1**	
	(v)	At (0,2) gradient is –1 so gradient of normal is 1 Normal is $y = x + 2$. $y = x+2,\ y = (x+2)e^{-x}$ $\Rightarrow 0 = (x+2)(1-e^{-x})$ $\Rightarrow x = -2\ (or\ 0)$ New intersection point is (-2,0).	B1 M1 A1 **3**	
	(vi)	Required area is $\int_1^3 (x+2)e^{-x}dx$ $= \left[-e^{-x}(x+2)\right]_1^3 + \int_1^3 e^{-x}dx$ $= \left[-e^{-x}(x+2)\right]_1^3 + \left[-e^{-x}\right]_1^3$ $= \frac{-6}{e^3} + \frac{4}{e}$	B1 M1 A1 A1 A1 **5**	or equivalent

9	**(i) (A)**		The transformation is a stretch with the x-axis invariant and of scale factor 2.	B1 B1 **2**	Same orientation y values doubled
	(i) (B)		The transformation is a reflection in the y-axis.	B1 B2 **3**	same shape Inversion
	(i) (C)		The transformation is a translation of 2 units parallel to the x-axis, ie $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	B1 B2 **3**	Same shape Moved 2 to the right
	(ii)	There is a set of values of y (for example, $y = 1$) for which there are three corresponding values of x (so an inverse would be multivalued).		B1 B1 **2**	
	(iii)			B1 **1**	
	(iv)	$\mathrm{f}(x) = x^2(x+2)$ $\Rightarrow \mathrm{f}'(x) = 3x^2 + 4x$ So the gradient at (1,3) is 7. The gradient on the inverse (which is a reflection of the original in $y = x$) is therefore $^{-1}/_{7}$.		M1 A1 M1 A1 **4**	
	(v)	The graph and its reflection must intersect on the axis of reflection, ie y = x, so solve $y = x,\ y = x^2(x+2)$ $\Rightarrow x = x^2(x+2)$ $\Rightarrow 0 = x(x^2 + 2x - 1)$ $\Rightarrow x = 0, -1 \pm \sqrt{2}$ The positive non-zero root is as given.		M1 M1 E1 **3**	