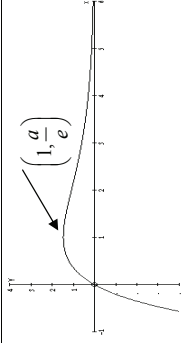


MEI STRUCTURED MATHEMATICS

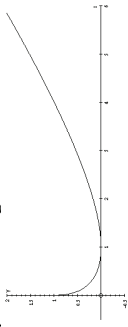
METHODS OF ADVANCED MATHEMATICS, C3

Practice Paper C3-A

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1	Product of two numbers, one of which is even is always even. Two consecutive numbers contain an even number. OR acceptable alternatives	B1 B1	
2	$y = \sqrt{1+x^3}$ Let $u = 1+x^3 \Rightarrow \frac{du}{dx} = 3x^2$ $y = u^{\frac{1}{2}} \Rightarrow \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2}u^{-\frac{1}{2}} \times 3x^2 = \frac{3x^2}{2\sqrt{1+x^3}}$	M1 A1 A1	Chain rule $\frac{dy}{dx}$ Answer
3	(i) Substitute: $0 = a \ln 2b \Rightarrow \ln 2b = 0 \Rightarrow 2b = 1 \Rightarrow b = \frac{1}{2}$ $1 = a \ln 2 \Rightarrow a = \frac{1}{\ln 2}$	M1 A1 A1	
	(ii) $\left \ln \left(\frac{1}{2}x \right) \right < 2 \Rightarrow \left \ln \frac{1}{2}x \right < 2 \ln 2$ $\Rightarrow -2 \ln 2 < \ln \frac{1}{2}x < 2 \ln 2$ $\Rightarrow \ln \frac{1}{4} < \ln \frac{1}{2}x < \ln 4$ $\Rightarrow \frac{1}{4} < \frac{1}{2}x < 4 \Rightarrow \frac{1}{2} < x < 8$	M1 M1 A1 A1	Modulus Powers of logs
4	(i) $y = axe^{-x} \Rightarrow \frac{dy}{dx} = ae^{-x} - axe^{-x} = ae^{-x}(1-x)$ $\frac{dy}{dx} = 0 \Rightarrow x = 1$ only at $\left(1, \frac{a}{e}\right)$ $\frac{d^2y}{dx^2} = -ae^{-x}(1-x) - ae^{-x}$; When $x = 1, \frac{d^2y}{dx^2} < 0$ \Rightarrow Maximum	M1 A1 M1 A1 B1	Product $= 0$ or any equivalent argument
	(ii) 	B1 B1	For curve for stationary point

5	$\int_2^3 xe^{2x} dx$ $u = x, \quad \frac{dv}{dx} = e^{2x}$ $\frac{du}{dx} = 1, \quad v = \frac{1}{2}e^{2x}$ $= \left[\frac{1}{2}xe^{2x} \right]_2^3 - \frac{1}{2} \int_2^3 e^{2x} dx = \left[\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} \right]_2^3$ $= \frac{5}{4}e^6 - \frac{3}{4}e^4 = 463.3$	Choice of u	M1 A1 M1 A1 A1	5
6	$\frac{d}{dx}(x \ln x) = \ln x + x \times \frac{1}{x} = \ln x + 1$ $\Rightarrow \int x \ln x = \int (\ln x + 1) dx = \int \ln x dx + x$ $\Rightarrow \int_2^3 \ln x dx = \left[x \ln x - x \right]_2^3 = (3 \ln 3 - 3) - (2 \ln 2 - 2)$ $= 3 \ln 3 - 2 \ln 2 - 1$ $= \ln \frac{27}{4} - 1$	Product Integrand limits	M1 A1 M1 A1 M1 A1	5
7	$(i) \quad x = 5 \sin \frac{\pi}{2} - 4 \cos \frac{\pi}{2} = 5$ $(ii) \quad \frac{dx}{d\theta} = 5 \cos \theta + 4 \sin \theta. \quad \text{When } \theta = \frac{\pi}{2}, \quad \frac{dx}{d\theta} = 4$ $\frac{dx}{d\theta} \times \frac{d\theta}{dt} = 4 \times 0.1 = 0.4$		B1 M1 A1 M1 E1	1 4 5
Section B				
8	$(i) \quad f(x) = \frac{x}{x^2+1} \Rightarrow f(x) = \frac{(x^2+1) \cdot 1 - x \cdot 2x}{(x^2+1)^2}$ $= \frac{1-x^2}{(x^2+1)^2}$	Formula Middle section answer	M1 A1 E1	3
	$(ii) \quad f(x) = \frac{1-x^2}{(x^2+1)^2} = 0 \Rightarrow 1-x^2 = 0 \Rightarrow x = \pm 1$ <p>When $x = 1$, $f(x) = \frac{1}{1+1} = \frac{1}{2}$ i.e. $\left(1, \frac{1}{2}\right)$</p> <p>Other stationary point is when $x = -1$, $f(x) = \frac{-1}{1+1} = -\frac{1}{2}$</p> <p>i.e. $\left(-1, -\frac{1}{2}\right)$</p>	Substitute Find $f(x)$	E1 B1	2
	<p>The graph is odd.</p> $f(-x) = \frac{-x}{(-x)^2+1} = -\frac{x}{x^2+1} = -f(x)$		B1 B1	2

	$(iv) \quad u = x^2 \Rightarrow du = 2x dx$ <p>When $x = 1, u = 1$</p> <p>When $x = 4, u = 16$</p> $\Rightarrow \int_1^4 \frac{x}{x^2+1} dx = \int_1^{16} \frac{1}{u+1} \cdot \frac{1}{2} du$ $\Rightarrow a = 1, b = 16, k = \frac{1}{2}$	M1 B1 B1 A1 A1	5
	<p>(v) Because the function is odd the area in $[-1,0]$ is equal in magnitude but opposite in sign to the area in $[0,1]$</p> <p>So shaded area = $\int_1^x \frac{x}{x^2+1} dx + 2 \int_0^x \frac{x}{x^2+1} dx$</p> $= \int_1^x \frac{1}{u+1} du + 2 \int_0^x \frac{1}{u+1} du$ $= \frac{1}{2} [\ln(u+1)]_1^x + 2 \left[\frac{1}{2} \ln(u+1) \right]_0^x$ $= \frac{1}{2} \ln \frac{17}{2} + 2 \cdot \frac{1}{2} \ln 2 = \frac{1}{2} \left(\ln \frac{17}{2} + \ln 4 \right) = \frac{1}{2} \ln 34 \approx 1.763$	B1 M1 A1 M1 A1 A1	6
9	$(i) \quad \sqrt{y} = 1 - \sqrt{x}$ $\Rightarrow y = (1 - \sqrt{x})^2 = 1 + x - 2\sqrt{x}$ <p>This is undefined for $x < 0$ but is defined for $x > 1$ so that part is missing.</p> 	M1 A1 A1 B1 B1 B1 B1	7
	<p>(ii) A is $\sqrt{-x} + \sqrt{y} = 1$ for $-1 \leq x \leq 0$</p> <p>B is $\sqrt{-x} + \sqrt{-y} = 1$ for $-1 \leq x \leq 0$</p> <p>C is $\sqrt{x} + \sqrt{-y} = 1$ for $0 \leq x \leq 1$</p>	B1 B1 B1 B1 B1 B1	6
	<p>(iii) Area in Fig. 9.1 is $\int_0^1 (1 + x - 2\sqrt{x}) dx$</p> $= \left[x + \frac{1}{2}x^2 - \frac{4}{3}x^{3/2} \right]_0^1 = 1 + \frac{1}{2} - \frac{4}{3} = \frac{1}{6}$ <p>\Rightarrow Area of shape is $\frac{1}{6} + \frac{1}{6} + 2 \times \frac{1}{6} = \frac{1}{3}$</p>	M1 A1 A1 M1 A1	5